- simple inequalities such as
  $n! > 2^n$
  $2^n > n^2$ $(n \geq 4)$
  $(1+x)^n \geq 1+nx$ $(x > -1)$

Induction Inequality Proofs

Prove $n! > 2^n$ $(n \geq 4)$

$n=4$ $\Rightarrow$ $4! = 24$, $2^4 = 16$ } yes $24 > 16$

Assume — Assume $k! > 2^k$

Prove true for $n=k+1$ — $\Rightarrow$ Is $(k+1)! > 2^{k+1}$?

$k!(k+1) > (2)2^k$

$k!\,5 > (2)2^k$ true

$K \geq 4$
$\Rightarrow K+1 \geq 5$

$\Rightarrow$ true for $n=4$, $n=k$, $n=k+1$
$\Rightarrow$ true for all $n \geq 4$

Q4 (a) Prove $5^n - 2^n$ is divisible by 3. $n \in \mathbb{N}$

$n=1$ — $\Rightarrow$ $5^1 - 2^1 = 3$ is divisible by 3

Assume — Assume $5^k - 2^k$ is divisible by 3

Prove true for $n=k+1$ — Is $5^{k+1} - 2^{k+1}$ divisible by 3

$F(k+1)$ $\quad (5)5^k - (2)\cancel{2}^k$
$- 2F(k)$ $\quad \mp(2)5^k \pm \cancel{(2)}2^k$

$3(5^k)$ which is divisible by 3

If $F(k+1) - 2F(k)$ is divisible by $3 \Rightarrow F(k+1)$ is also

$\Rightarrow F(k+1)$ is divisible by 3.

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