Trigonometry

Trigonometry is the study of triangles. However concepts are also used in Complex Numbers, Calculus and The Line and Circle.

	Radians	Degrees
Length of Arc	$r\theta$	$2\pi r \times \frac{\theta}{360}$
Area of Sector	$\frac{1}{2}\theta r^2$	$\pi r^2 \times \frac{\theta}{360}$

Solving Triangles

Tan, Sin or **Cos** $\left(T = \frac{O}{A} \quad S = \frac{O}{H} \quad C = \frac{A}{H}\right)$

Used to solve for sides and angles in right angled triangles.

Pythagoras $(H^2 = A^2 + O^2)$

To find 3rd side of a right angled triangle when we have the other two.

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\,cosA$$

To find the 3rd side of a triangle when we have the other two and the angle between them **OR** to find the angle when given the 3 sides of the triangle.

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Need one side and an angle opposite as well as one other angle or side.

Area of Triangle

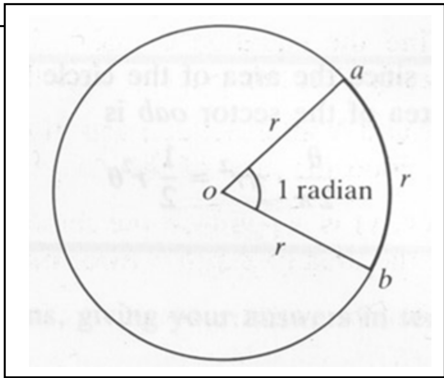
$$\frac{1}{2}ab\sin C$$

Half the product of any two sides multiplied by the sine of the angle between them.

Radian Measure

$Degrees\ to\ radians\ \times \frac{\pi}{180}$

$Radians\ to\ degrees\ \times \frac{180}{\pi}$



Convert 30° in terms of π

$$30° = \frac{30\pi}{180} = \frac{\pi}{6}$$

Convert $\frac{\pi}{3}$ into degrees

$$\frac{\pi}{3}.\frac{180}{\pi} = 60°$$

Convert 1.5 radians into degrees

$$1.5 \times \frac{180}{\pi} = 1.5 \times \frac{180}{3.14} = 86°$$

Convert 60° into radians

$$60° = 60 \times \frac{\pi}{180} = 60 \times \frac{3.14}{180} = 1.05\ radians$$

Trig Identities

Use tables to prove 8 identities (see proofs handout)

Eg.

$\cos 2A = \cos^2 A - \sin^2 A$

Must be able to use all 24 identities

Inverse Trigonometric Functions

When $\sin^{-1} x = A$ then $\sin A = x$

When $\cos^{-1} x = A$ then $\cos A = x$

When $\tan^{-1} x = A$ then $\tan A = x$

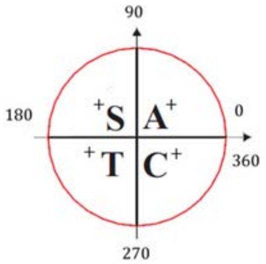
Domain and Range Inverse Graphs

$\sin^{-1} x$ – Domain $[-1,1]$ Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1} x$ – Domain $[-1,1]$ Range $[0, \pi]$

$\tan^{-1} x$ – Domain $[-\infty, \infty]$ Range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Unit Circle



The coordinates of any point on the unit circle are $(cos\ \theta, sin\ \theta)$

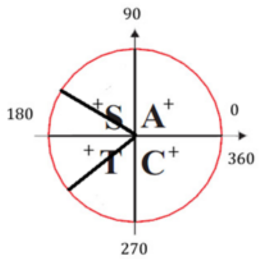
Trigonometric Equations

Between 0° and 360° there may be two angles with the same trigonometric ratio.
Eg $\cos 120° = -\frac{1}{2}$ and $\cos 240° = -\frac{1}{2}$

To solve trigonometric equation do the following:

1. Ignore the sign and calculate the related angle
2. From the sign decide which quadrants the angles lie.
3. Using a rough diagram state the angles.

$\boldsymbol{\cos\theta = -\frac{\sqrt{3}}{2}}$ **where** $\boldsymbol{0 \le \theta \le 360°}$



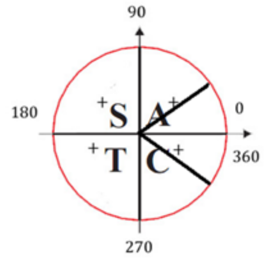
cos is negative in the 2nd and 3rd quadrant.

For reference angle use tables to check the *cos* of which angle gives $\frac{\sqrt{3}}{2}$
$\theta = 30°$

Required angles are $180 - 30 = 150°$ and $180 + 30 = 210°$

$\boldsymbol{\cos 2A = \frac{\sqrt{3}}{2}}$ **where** $\boldsymbol{0 \le A \le 2\pi}$

$\cos 2A$ is positive therefore the answers are in the 1st and 4th quadrants.



Reference angle = 30

$2A = 30$
$A = 15$

$2A = 360 - 30 = 330$
$2A = 360 - 30 = 115$

To find the answers for 2A we must keep adding 360 and divide by two to get A until the answers we get for A are outside the boundaries given in the question.

$2A = 30, 390$
$A = 15, 185$

$2A = 360 - 30 = 330, 690$
$A = 115, 345$

Sum, Difference and Product Formula

Express $\cos 3A \sin A$ as a sum or difference

Use formulae in tables

$\cos 3A \sin A = \frac{1}{2}(2\cos 3A \sin A)$

$= \frac{1}{2}(\sin 4A - \sin 2A)$

Can be useful for Integration questions

Tackling Problems in Trigonometry

1. Always draw a triangle. Put in all the information you can.
2. If two or more triangles linked draw them separately.
3. Watch out for common values. We can carry common from one triangle to another.
4. If right angled triangle use sin, cos, tan and Pythagoras
5. If not right angled use the Sine or Cosine Rule and area of triangle formula as needed.

3D Triangles
Redraw each triangle separately. Find common sides and apply Pythagoras, Cosine and Sine rules.