Example 5

(i) Evaluate $^{10}P_3$ (ii) Find n if $7[^nP_3] = 6[^{n+1}P_3]$

(i) $^{10}P_3 = 720$

(ii) $7[^nP_3] = 6[^{n+1}P_3]$

$^nP_r = \frac{n!}{(n-r)!}$

$7[\cancel{n} \times \cancel{(n-1)} \times (n-2)] = 6[(n+1) \times \cancel{n} \times \cancel{(n-1)}]$

$7(n-2) = 6(n+1)$

$7n - 14 = 6n + 6$

$n = 20$

Example 6

How many different four-letter arrangements can be made from the letters of the word THURSDAY if a letter cannot be repeated in an arrangement?
How many of the arrangements begin with the letter D and end with a vowel?

(i) $\boxed{8} \times \boxed{7} \times \boxed{6} \times \boxed{5} = {}^8P_4 = 1680$

(ii) $\boxed{1} \times \boxed{6} \times \boxed{5} \times \boxed{2} = 60$

8. How many different arrangements can be made from the letters of the word PROBLEM?

(i) How many of these arrangements begin with a vowel?

(ii) In how many of these arrangements do the two vowels come together?

$\boxed{7} \times \boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1} = 7!$

$7! = 5040$

Vowel first (i)

$\boxed{2} \times \underbrace{\boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1}}_{\text{leftover}}$

Vowel

P R B L M (ii) Treating vowels as 1 letter × arrangements

OE

$(6!) \times (2) = 1440$

26. A competition has a first prize, a second prize and a third prize. 10 competitors enter this competition and the 3 prizes are awarded in order of merit.

(i) Find the number of different ways in which these prizes could be won.

Smith and Jones are 2 of the 10 competitors. Find the number of different ways in which the prizes could be won if

(ii) neither Smith nor Jones wins a prize

(iii) each of Smith and Jones wins a prize.

Possible ordered ways to select 3 out of 10 choices (i)

${}^{10}P_3 = 720$

OR $\frac{10!}{7!} = 720$

OR $\boxed{10} \times \boxed{9} \times \boxed{8} = 720$

neither Smith or Jones (ii)

$\boxed{8} \times \boxed{7} \times \boxed{6} = 336$

OR ${}^{8}P_3 = 336$

(iii)

$\boxed{8}$ How many winners possible?

$\boxed{A\ B\ C}$ 6 arrangements

$8 \times 6 = 48$

$\left(\underset{J}{\boxed{1}} \times \underset{S}{\boxed{1}} \times \underset{\text{other}}{\boxed{8}}\right) \times \left(\underbrace{\boxed{3} \times \boxed{2} \times \boxed{1}}_{\text{arrangements}}\right)$