Probability 1

Chapter 1

Section 1.5 Mutually exclusive events – The addition rule

PROJECT MATHS
Text & Tests 5
LEAVING CERTIFICATE HIGHER LEVEL
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PROBABILITY & STATISTICS

23

Consider the following two events when drawing a card from a pack of 52 playing cards:

A = drawing an ace B = drawing a king.

These two events are said to be **mutually exclusive** as they cannot occur together.

If the events A and B cannot happen together, then

$$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

This is called the **addition law** for mutually exclusive events.

> Outcomes are mutually exclusive if they cannot happen at the same time.

So $P(\text{draw an ace } \textbf{or} \text{ king}) = P(\text{ace}) + P(\text{king})$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{8}{52} = \frac{2}{13}$$

When events are not mutually exclusive

We will now consider events which may occur at the same time.

If A is the event: selecting an ace from a pack of cards and

B is the event: selecting a heart from a pack of cards

then $P(\text{A}) = \frac{4}{52}$ and $P(\text{B}) = \frac{13}{52}$

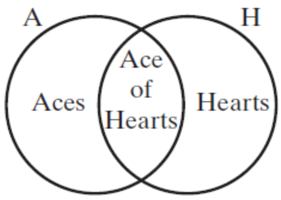
In this situation, both events may occur at the same time since the *ace of hearts* is common to both.

In general, when two events A and B can occur at the same time,

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$$

Thus in the example given above,

$$P(\text{ace or heart}) = P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart})$$
$$= \tfrac{4}{52} + \tfrac{13}{52} - \tfrac{1}{52}$$
$$= \tfrac{16}{52}$$



This result can be verified as there are 4 aces and 13 hearts in a pack of cards. Since one of the aces is the ace of hearts, there are 16 aces or hearts in the pack.

i.e. $P(\text{ace or a heart}) = \frac{16}{52}$, as already found.

Example 1

A card is drawn at random from a pack of 52.

What is the probability that the card is

(i) a club (ii) a king (iii) a club or a king

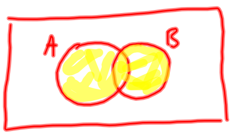
(iv) a red card (v) a queen (vi) a red card or a queen

(i) $P(\text{club}) = \frac{1}{4}$

(ii) $P(\text{King}) = \frac{1}{13}$

It can be both K and club

$\Rightarrow$ not M.E.



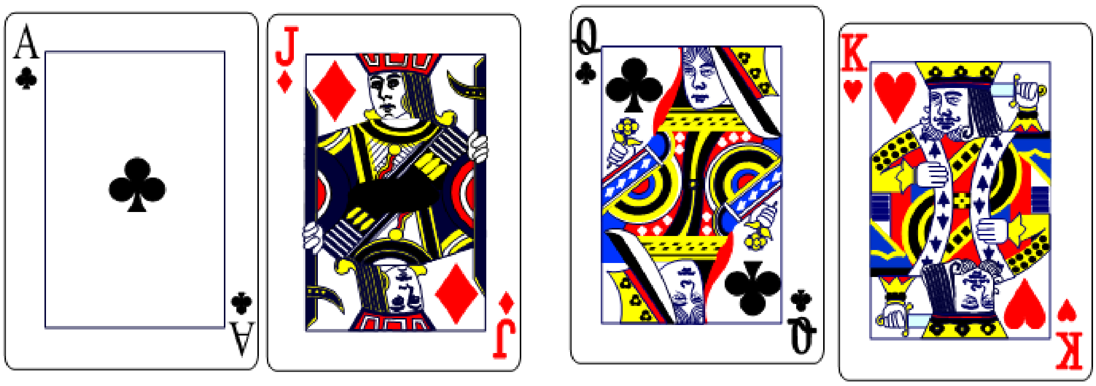
$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

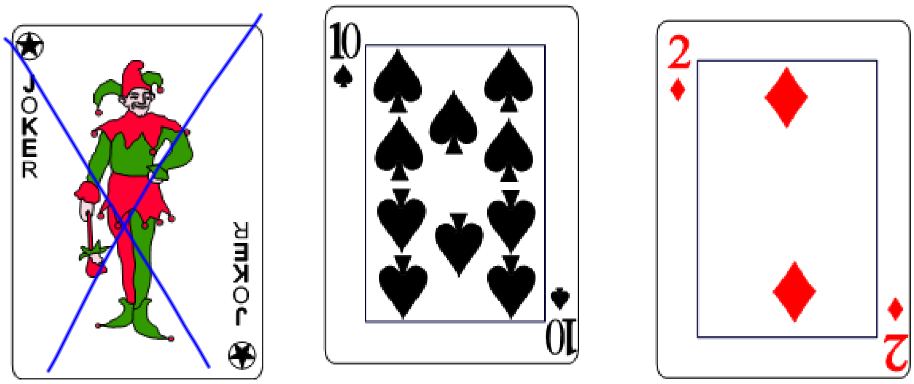
$P(\text{King and Club}) = \frac{1}{52}$

(iii) $P(\text{King or club}) = \frac{1}{13} + \frac{1}{4} - \frac{1}{52} = \frac{4}{13}$

(iv) $P(\text{Red}) = \frac{1}{2}$ (v) $P(Q) = \frac{1}{13}$

(vi) $P(\text{Red or queen}) = \frac{1}{2} + \frac{1}{13} - \frac{1}{52} = \frac{29}{52}$

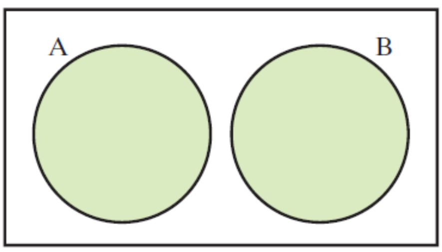




52 Cards
26 Red
26 Black
13 Hearts
3 Picture cards in each suit

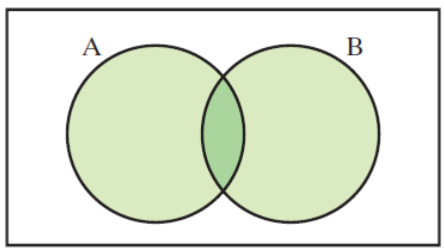
Venn diagrams for mutually exclusive events

(i) Mutually exclusive



$P(\text{A or B}) = P(\text{A}) + P(\text{B})$
$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B})$

(ii) Non-mutually exclusive



$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$
$P(\text{A} \cup \text{B}) = P(\text{A}) + P(\text{B}) - P(\text{A} \cap \text{B})$

Example 2

A and B are two events such that $P(\mathrm{A}) = \frac{19}{30}$, $P(\mathrm{B}) = \frac{2}{5}$ and $P(\mathrm{A} \cup \mathrm{B}) = \frac{4}{5}$.
Find $P(\mathrm{A} \cap \mathrm{B})$.