Probability 1

chapter 1

Section 1.6 The multiplication law for independent events

PROJECT MATHS
Text & Tests 5
LEAVING CERTIFICATE
HIGHER LEVEL
STRAND 1
PROBABILITY & STATISTICS

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Paul spins a coin  and rolls a dice.

His results are shown on the right.

The coin and the dice do not affect each other, so their outcomes are **independent**.

There are 12 equally likely outcomes of the coin and dice, as shown in the diagram on the right.

From the sample space, we can see that the probability of a head and a 5 is $\frac{1}{12}$.

The probability of each outcome can also be found by multiplying the separate probabilities, as shown above.

Dice		H(ead)	T(ail)
	6	H, 6	T, 6
	5	H, 5	T, 5
	4	H, 4	T, 4
	3	H, 3	T, 3
	2	H, 2	T, 2
	1	H, 1	T, 1

Coin

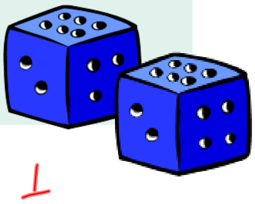
This illustrates the **multiplication law** of probability which states that for independent events A and B,

$$P(A \text{ and } B) = P(A) \times P(B)$$

This law is sometimes called the AND Rule.

The multiplication law applies to any number of independent events.

Example 1



When two dice are thrown, what is the probability of getting
(i) two sixes (ii) 4 or more on each die?

$P(A \cap B) = P(A) \times P(B)$

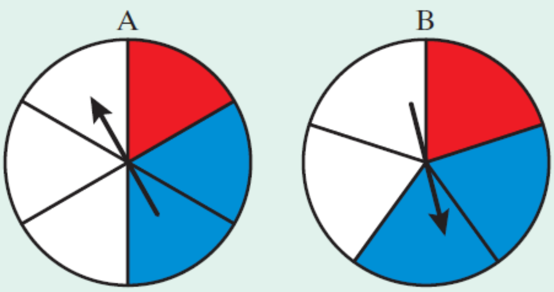
$P(2\text{ sixes}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(4\text{ or more}) = \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

Example 2

These two spinners are spun.
What is the probability that
(i) spinner A shows red
(ii) spinner B shows red
(iii) both spinners show red
(iv) A shows red and B shows blue
(v) both show blue
(vi) both show white
(vii) neither shows white?



(i) spinner A shows red $= 1/6$
(ii) spinner B shows red $= 1/5$
(iii) both spinners show red $= (1/6)(1/5) = 1/30$
(iv) A shows red and B shows blue $= (1/6)(2/5) = 1/15$
(v) both show blue $= (2/6)(2/5) = 2/15$
(vi) both show white $= (1/2)(2/5) = 2/10$
(vii) neither shows white? $=$ P(A not white) and P(B not white)
$= (1/2)(3/5) = 3/10$

Example 3

A gambler must throw a 6 with a single dice to win a prize.
Find the probability that he wins at his third attempt. [loosing first & 2nd attempt]

If he wins on 3rd attempt
⇒ he looses in first & 2nd go.

$$\Rightarrow P(\text{not } 6, \text{not } 6, 6)$$
$$= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$$
$$= \frac{25}{216}$$

Example 4

Three pupils A, B and C have their birthdays in the same week.
What is the probability that the three birthdays
(i) fall on a Monday
(ii) fall on the same day
(iii) fall on three different days?

$P(m) = \frac{1}{7}$

(i) $P(M, M, M) = \left(\frac{1}{7}\right)\left(\frac{1}{7}\right)\left(\frac{1}{7}\right) = \frac{1}{343}$

* (ii) $P(\text{all M or T or W or T or F or S or S}) = \frac{1}{343}+\frac{1}{343}+\frac{1}{343}+\frac{1}{343}+\frac{1}{343}+\frac{1}{343}+\frac{1}{343}$

$= \frac{7}{343} = \frac{1}{49}$

note book has a different approach.

(iii) $P(\text{different}) = \underbrace{1}_{\text{A on any day}}\underbrace{\left(\frac{6}{7}\right)}_{\text{B on any other day}}\underbrace{\left(\frac{5}{7}\right)}_{\text{C on any other day}} = \frac{30}{49}$