

chapter

7

## Algebra 3

## Section 7.5 Proofs of abstract inequalities

PROJECT MATHS

Text &amp; Tests 6

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Must know

•  $x^2 \geq 0$

•  $(a+b)^2 = a^2 + 2ab + b^2 \geq 0$   
 $\Rightarrow a^2 + b^2 \geq -2ab$

•  $(a-b)^2 = a^2 - 2ab + b^2 \geq 0$   
 $a^2 + b^2 \geq 2ab$

If  $a$  and  $b$  are real numbers, then

$$a^2 \geq 0 \text{ and } b^2 \geq 0$$

$$(a + b)^2 \geq 0$$

$$(a - b)^2 \geq 0$$

$$-(a + b)^2 \leq 0$$

$$-(a - b)^2 \leq 0$$

### Example 1

Prove that  $a^2 + b^2 \geq 2ab$  for all  $a, b \in \mathbb{R}$ .

$$a^2 + b^2 \geq 2ab$$

$$a^2 - 2ab + b^2 \geq 0$$

$$(a - b)^2 \geq 0 \quad \text{true}$$

### Example 2

If  $a > 0$  and  $b > 0$ , prove that  $\frac{a}{b} + \frac{b}{a} \geq 2$ .

$$a^2 + b^2 \geq 2ab \quad (\times ab)$$

$$a^2 - 2ab + b^2 \geq 0$$

$$(a - b)^2 \geq 0 \quad \text{true}$$

**Example 3**

Show that  $x^2 + 4x + 6 > 0$  (i.e. is positive) for all  $x \in \mathbb{R}$ .

$$\begin{array}{c} x^2 + 4x + 6 > 0 \\ \begin{array}{c} x \quad 2 \\ \begin{array}{|c|c|} \hline x^2 & 2x \\ \hline 2x & 4 \\ \hline \end{array} \end{array} \end{array} \quad x^2 + 4x + 4 = (x+2)^2$$

$$x^2 + 4x + 6 = (x+2)^2 + 2 \geq 2 > 0 \\ \Rightarrow .$$

**Example 4**

Show for all real numbers  $a, b > 0$  that  $(a+b)\left(\frac{1}{a} + \frac{1}{b}\right) \geq 4$ .

$$\begin{aligned} \frac{(a+b)(b+a)}{ab} &\geq 4 \\ (a+b)(b+a) &\geq 4ab \\ a^2 + 2ab + b^2 &\geq 4ab \\ a^2 - 2ab + b^2 &\geq 0 \\ (a-b)^2 &\geq 0 \quad \text{true} \end{aligned}$$