4. If $a > 0$ and $b > 0$, show that

(i) $a + \frac{1}{a} \geqslant 2$ (ii) $\frac{1}{a} + \frac{1}{b} \geqslant \frac{2}{a+b}$.

(×a) $a^2 + 1 \geq 2a$

$a^2 - 2a + 1^2 \geq 0$

$(a-1)^2 \geq 0$

single fraction $\frac{b+a}{ab} \geq \frac{2}{a+b}$

(× LCD) $(a+b)(b+a) \geq 2ab$

$(a+b)^2 \geq 2ab$

$(a+b)^2 - 2ab \geq 0$

$a^2 + \cancel{2ab} + b^2 - \cancel{2ab} \geq 0$

$a^2 + b^2 \geq 0$ true

5. Prove that $a^2 - 6a + 9 + b^2 \geqslant 0$ for all real values of a and b.

notice :

$(a-3)^2$

$= a^2 - 6a + 9$

$\Rightarrow$ $a^2 - 6a + 9 + b^2 \geq 0$

$(a-3)^2 + b^2 \geq 0$

true

$(a+b)^2$

$= a^2 + 2ab + b^2$

$(a-b)^2$

$= a^2 - 2ab + b^2$

9. Factorise $a^3 + b^3$.
Hence prove that $a^3 + b^3 > a^2b + ab^2$ for all real $a > 0$ and $b > 0$.

$$a^3+b^3=(a+b)(a^2-ab+b^2)^{*}$$ this must be learnt!

$-a^2b-ab^2$ $\Rightarrow$

$$a^3+b^3-a^2b-ab^2>0$$
$$(a+b)(a^2-ab+b^2)-ab(a+b)>0$$
$$(a+b)(a^2-ab+b^2-ab)>0$$
$$(a+b)(a^2-2ab+b^2)>0$$
$$(a+b)(a-b)^2>0$$
true

factorise

OR thanks to Sineed!

$$\cancel{(a+b)}(a^2-ab+b^2)>\cancel{(a+b)}ab$$
$$a^2-2ab+b^2>0$$
$$(a-b)^2>0$$
true