

Chapter 7 Algebra 3

Section 7.7 Exponential equations

PROJECT MATHS Text & Tests 6

244

Example 1

Solve these equations. (i) $\frac{1}{8^x} = 16^{\frac{1}{3}}$ (ii) $27^{x-3} = 3 \times 9^{x-2}$

$$\Rightarrow 8^{-x} = 16^{\frac{1}{3}}$$

$$\begin{array}{l} 2^2=4 \\ 2^3=8 \\ 2^4=16 \end{array}$$

$$(2^3)^{-x} = (2^4)^{\frac{1}{3}}$$

$$2^{-3x} = 2^{\frac{4}{3}}$$

$$\Rightarrow -3x = \frac{4}{3}$$

$$x = -\frac{4}{9}$$

$$\Rightarrow 8^{-x} = 16^{\frac{1}{3}}$$

$$n = \log_{\boxed{8}} \boxed{16^{\frac{1}{3}}} = \frac{4}{9}$$

$$\Rightarrow x = -\frac{4}{9}$$

$$\begin{array}{l} 3^2=9 \\ 3^3=27 \end{array}$$

$$(3^3)^{x-3} = 3(3^2)^{x-2}$$

$$3^{3x-9} = (3)(3^{2x-4})$$

$$3^{3x-9} = 3^{2x-3}$$

$$\Rightarrow 3x-9 = 2x-3$$

$$x = 6$$

Example 2

If $y = 3^x$, express 3^{2x} in terms of y .

Hence solve the equation $3^{2x} - 4 \cdot 3^x + 3 = 0$.

$$y = 3^x \qquad 3^{2x} = (3^x)^2 = y^2$$

Rewrite $y^2 - 4y + 3 = 0$

$$(y - 3)(y - 1) = 0$$

$$y = 3 \quad | \quad y = 1$$

$$y = 3^x \Rightarrow 3 = 3^x \Rightarrow x = 1$$

$$\text{or } 1 = 3^x \Rightarrow x = 0$$

7. By letting $3^x = y$, solve the equation $3^{2x} - 12(3^x) + 27 = 0$.

$$y = 3^x \Rightarrow 3^{2x} = (3^x)^2 = y^2$$

Rewrite equation:

$$y^2 - 12y + 27 = 0$$

$$(y - 9)(y - 3) = 0$$

$$y = 9, \quad y = 3$$

$$y = 3^x \qquad 9 = 3^x \Rightarrow x = 2$$

$$3 = 3^x \Rightarrow x = 1$$