Example 3

Given that the intensity of an earthquake is represented by the formula $A = 10^M$, and the energy released during a quake by the formula $E \cong 10^{1.5M + 4.8}$, where A is the amplitude and M is the magnitude on the Richter scale, compare
(i) the intensity (ii) the energy of an earthquake of magnitude 6.1 on the Richter scale with a quake of magnitude 4.7.

$M = 6.1$

$A = 10^{6.1} = 1,258,925.4 \approx 1.2 \times 10^6$

$E = 10^{1.5(6.1)+4.8} = 8.9 \times 10^{13}$

$M = 4.7$

$A = 10^{4.7} \approx 5.0 \times 10^4$

$E = 10^{1.5(4.7)+4.8} = 7.0 \times 10^{11}$

COMPARING

Amplitudes

$\frac{1.2 \times 10^6}{5.0 \times 10^4} = 24$

$24 : 1$

Energy

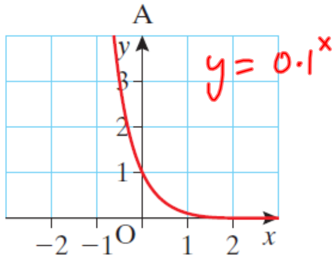
$\frac{8.9 \times 10^{13}}{7.0 \times 10^{11}} \approx 127$

$127 : 1$

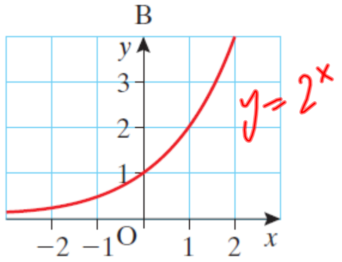
Exercise 7.8

1. Match each of the following exponential functions with one of the graphs.
   (i) $y = 2^x$ Increasing with $x=0, y=1$
   (ii) $y = (0.1)^x$ Decreasing
   (iii) $y = 10^x$ Increasing sharply
   (iv) $y = (0.5)2^x$ Increasing with $x=0, y=\frac{1}{2}$

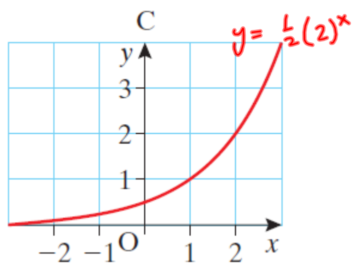
A



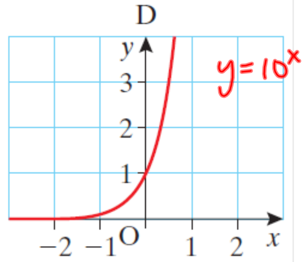
B



C



D



7. Carbon-14, the radioactive element of carbon, decays according to the formula $P = 100(0.99988)^n$, where P is the percentage of the original mass of Carbon-14 that remains after n years.
   (a) Find the percentage of Carbon-14 that remains after (i) 200 years (ii) 500 years.
   (b) Estimate (using trial and error) how long it will take the Carbon-14 sample to decay to half its original mass. Give your answer correct to the nearest 10 years.
   (c) A bone containing 79% of its original Carbon-14 was discovered in a bog in County Offaly. Estimate its age.

$P = 100(0.99988)^n \%$

(a) $n = 200 \Rightarrow P = 100(0.99988)^{200}$
$= 97.6\%$

$n = 500 \Rightarrow P = 100(0.99988)^{500}$
$= 94.7\%$

Half-life?
Trial & error

(b) $n = 2000 \Rightarrow P = 100(0.99988)^{2000}$
$P = 78.7\%$

$n = 10\,000 \Rightarrow P = 100(0.99988)^{10\,000}$
$= 30.1\%$

$n = 5780 \Rightarrow P \simeq 49.7\%$

exact value using logs

$n = \log_{0.99988} 0.5 = 5776$

$n = \log_{0.99988} 0.79 = 1964$