Functions

chapter 1

Section 1.2 Composition of functions

"function of a function"

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"g after f" $= g \circ f = gf(x)$

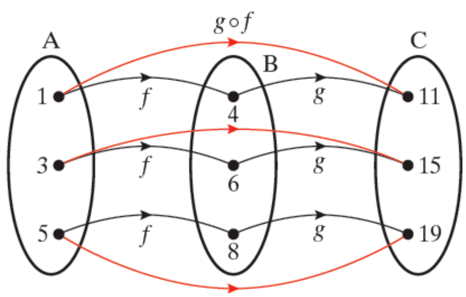
The diagram below shows a function *f* illustrated by the mapping diagram from A to B, and the function *g* illustrated by the mapping diagram from B to C.

The red arrows represent the couples of a new function combining *f* and *g*.

It is called the **composite function** *g* after *f*.
It is written as $g \circ f$, or simply ***gf***.

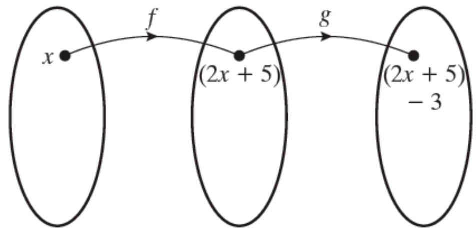
$g \circ f$ is read '*g* after *f*'.

The couples $g \circ f$ from the diagram are $\{(1, 11), (3, 15), (5, 19)\}$.



We will now consider two functions, $f(x) = 2x + 5$ and $g(x) = x - 3$, to find the rule for the composite function $gf(x)$.

These functions are illustrated on the right. In f, the output is $(2x + 5)$ when the input is x.



We will now use $(2x + 5)$ as the input for the function g.

Since $g(x) = x - 3$, then $gf(x) = g(2x + 5)$

$$= (2x + 5) - 3 \quad \text{... replacing } x \text{ with } (2x + 5)$$

$$gf(x) = 2x + 2$$

We will now consider what happens when the order of the functions is changed.

$$fg(x) = f(x - 3)$$

$$= 2(x - 3) + 5 \quad \text{... replacing } x \text{ with } (x - 3)$$

$$fg(x) = 2x - 1$$

Since $2x + 2 \neq 2x - 1$, this shows that $gf(x) \neq fg(x)$.

* In general, if f and g are two functions, then $fg(x) \neq gf(x)$.

Example 1

Given that $f(x) = x + 3$ and $g(x) = x^2 - 1$, find
(i) $fg(2)$ (ii) $gf(-1)$ (iii) $fg(x)$ (iv) $gf(x)$.
Find also the value of x for which $fg(x) = gf(x)$.

(i) $f[g(2)] = f[2^2-1] = f(3) = 3+3 = 6$ ✓

(ii) $g[f(-1)] = g[-1+3] = g(2) = 2^2 - 1 = 3$ ✓

(iii) $f[g(x)] = f[x^2-1] = [x^2-1]+3 = \boxed{x^2+2}$ ✓

(iv) $g[f(x)] = g[x+3] = (x+3)^2 - 1 = x^2+6x+9-1$

$= \boxed{x^2+6x+8}$ ✓

(v) $fg(x) = gf(x)$

$\Rightarrow \cancel{x^2} + 2 = \cancel{x^2} + 6x + 8$

$\rightarrow 6x + 6 = 0$

$6x = -6$

$x = -1$

Note: $f \circ f(x)$ is written as $f^2(x)$.

Example 2

Given $f(x) = 2x + 1, g(x) = \frac{1}{x}$ and $h(x) = x^2 - 4$, find

(i) $gf(x)$ (ii) $f^2(x)$ (iii) $gh(x)$ (iv) $h^2(x)$

(v) $hfg(x)$ (vi) the values of x for which $gh(x) = \frac{1}{12}$.

(ii) $f^2(x) = f[f\,x] = f\,(2x+1)$

$$= 2\,(2x+1) + 1$$

$$= 4x + 2 + 1$$

$$= 4x + 3$$

(iv) $h^2(x) = h[h(x)] = h\,[x^2 - 4]$

$$= (x^2 - 4)^2 - 4$$

$$= x^4 - 8x^2 + 16 - 4$$

$$= x^4 - 8x^2 + 12$$